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THE TRANSIENT LOAD SUPPORTING CAPACITY OF FLUID FILMS

(FIRST REPORT)

. . . *By* . . .

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THE TRANSIENT LOAD SUPPORTING CAPACITY OF FLUID FILMS

by

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A study is presented in this paper of the thickness and the load supporting capacity of fluid films entrapped between rotating circular discs, under transient conditions. Part one, (see Fig. 1) is devoted to a theoretical analysis of the thickness and the load supporting capacity of fluid films under steady state conditions. Part two presents the results of an experimental investigation of these thicknesses and loads under transient conditions.

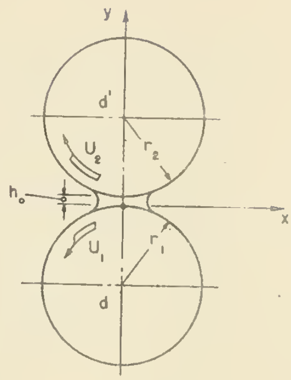

FIG. 1. DISCS POSESSING ROTARY BUT NO APPROACH MOTION WITH ENTRAPPED FLUID WEDGE.

Car ful instrumentation of this very complex non-steady-state problem, has yielded solutions which have not been obtainable by theoretical studies, except for cases involving assumptions which greatly alter the true results. It is found that the approach motion of the surfaces bounding the film is responsible for a much greater part of its load supporting

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action than is the steady state rotational motion.

STATEMENT OF PROBLEM:

The main problem is the investigation of the thickness and the corresponding load supporting capacity of fluid films entrapped between circular rotating discs on parallel shafts which have their center-to-center distance changed so as to cause the film to become alternately thin then thick.

METHOD OF ATTACK:

An exact analytical solution to the steady state problem was secured (see Part one). The author had previously obtained an approximate solution (2).

The transient state problem was solved by experimental means; it is much too comolicated to solve by analytical methods if one retains such factors as the deformation of the surfaces of the discs, surface roughness, etc. These factors occur in their natural way in the experimental solution.

DESCRIPTION OF THE EXPERIMENTAL SET-UP (3):

Two parallel steel shafts (see Fig. 2E) were prepared and mounted with adjustable taper roller bearings in hangers, one of which was attached to the upper beam with a strong-back with an adjusting screw for controlling the initial gap between discs. The other shaft was mounted in adjustable roller bearings in hangers secured to the lower beam which

(2) See Bibliography
(3) See Fig. 1E

could be vibrated transversely by means of a Westinghouse Fatigue motor. The frequency and amplitude of vibration were controlled by an oscillator and an amplifier. No lost motion was permitted anywhere. Immediate detection of lost motion was available. Each shaft was driven by a separate air turbine wheel mounted on the shaft. A D.C. excited coil was placed in the space provided on the lower shaft. A pick-up coil was arranged on the upper shaft. These coils did not revolve with the shafts. As the lower beam vibrated a signal occurred in the pick-up coil. This signal was fed directly to a galvanometer type oscillograph fitted with an automatic camera which gave permanent records of the signal generated. Oscillograms were taken for cases of the discs revolving with no oil on their surfaces. A certain height of wave form on the oscillogram results. Then, another oscillogram was secured without changing a setting of any kind, but permitting oil to flow onto the surface of the lower discs on the oncoming side. The second oscillogram showed a much reduced amplitude of vibration of the beam. The thickness of the film and its load could then be determined by methods outlined later. Several sets of such records were taken for different settings of the amplifier.

Lead wires were rolled between the discs a sufficient number of times to insure no "spring back" of the thickness of the leads with the lower beam supported by jacks to

prevent bending. The thickness of these leads, determined by interferometer methods, gave the neutral gap thickness h_o. Other leads were rolled between the discs and pounded by the vibrator until the height of the scope wave form reached that which it had when the gap contained air. These lead thicknesses gave the amplitude of vibration of the lower shaft discs for various power inputs to the vibration motor coils and the pick-up excitation-coil.

The pick-up apparatus showed a nearly linear response for these small amplitudes of vibration (0.001 approx.). The dynamic spring constant of the beam was secured. Thus there resulted a means of obtaining a continuous picture of the thickness of the film and its load supporting capacity under these transient conditions.

The existing experimental set-up could not be used to obtain a steady state solution of the problem, since the pick-up apparatus did not respond well to A.C. excitation. A relatively large neutral gap setting was chosen, which, according to existing theory, would yield almost no steady state load supporting component. Since there is little steady state load for the particular chosen neutral gap setting, the beam may be considered as vibrating about its neutral position.

NOMENCLATURE:

a = pole distance

α = abscissa of the bi-polar coordinate system, pressure angle

β = ordinate of the bi-polar coordinate system

Ψ = stream function

d = distance from the origin to the center of the disc in the bi-polar solution

$$G = \frac{12 \mu_a \; U \sqrt{2rh_o}}{h_o^2}$$

h^2 = reciprocal of "stretch factor"

h_o = neutral gap thickness between discs

p = pressure

P = load-supporting capacity of the film per unit face width, pitch point

$$r = \frac{r_1 r_2}{r_1 + r_2}$$

r_1 and r_2 = radii of curvature of the discs

$$S = \tan^{-1} \frac{x}{\sqrt{2 rh_o}}$$

T = torque

U_1 and U_2 = peripheral velocities of rotating discs

$$U = \frac{U_1 + U_2}{2}$$

u, v, and w = coordinate velocity components

x_o = effective width of band of contact

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ = Laplacian operator

μ_a = viscosity at atmospheric pressure and stated temperature

μ = viscosity

ω = angular velocity

ρ = density

$$\theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$\chi = hV$ by definition

INTRODUCTION:

For some time the author has been aware of the fact that the transient state of affairs, in many hydrodynamical problems, accounts for the relatively large load supporting capacity of fluid films. Most of our practical engineering applications, involving fluid films such as the lubrication of various bearings and gear teeth, come under this category. However, these particular non-steady-state problems are, at present, wholly unsolvable by analytical means, at least in their entirety.

It has been only recently that we at the Naval Postgraduate School have acquired accurate and delicate enough equipment to undertake the solution of such a complex problem. Part two of this report presents the results of an investigation of the load supporting capacity of two oils, namely, Navy 5190 and 2190T. The results have been limited by the capacity of our equipment, but they show clearly that

relatively thin oil film loads and thicknesses can and have been measured by these methods. Modifications are already under way which will permit the investigation of heavier film loads.

We now focus our attention on the analytical solution of the steady state problem. The bi-polar coordinate system is conveniently employed, since it provides boundaries of exactly the nature required in this investigation.

PROCEDURE:

The general procedure followed in the solution of this problem is to (a) write the equations of viscous fluid motion in the Cartesian coordinate system, (b), assume that the fluid is incompressible and that constant-viscosity-steady-state-conditions exist, (c) accomplish the transformation to the bi-polar coordinate system, (d) obtain the expressions which indicate the characteristics of the fluid film.

EQUATIONS OF MOTION FOR STEADY STATE CONSTANT VISCOSITY INCOMPRESSIBLE FLUID CONDITIONS

Fig. 2 shows an elemental cube of the fluid with its indicated positive stresses. Texts on hydrodynamics present the following equations for viscous fluid motion in which the viscosity μ is considered constant.

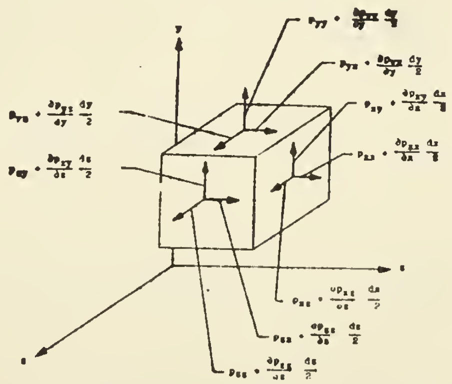


Fig. 2 Elementary Cube of Oil With Indicated Positive Stresses

They are

$$\left.\begin{aligned}\rho \frac{Du}{Dt} &= \rho X - \frac{\partial p}{\partial x} + \frac{1}{3}\frac{\partial \theta}{\partial x} + \mu \nabla^2 u \\ \rho \frac{Dv}{Dt} &= \rho Y - \frac{\partial p}{\partial y} + \frac{1}{3}\frac{\partial \theta}{\partial y} + \mu \nabla^2 v \\ \rho \frac{DW}{Dt} &= \rho Z - \frac{\partial p}{\partial z} + \frac{1}{3}\frac{\partial \theta}{\partial z} + \mu \nabla^2 W\end{aligned}\right\} \text{---} (1)$$

If the body forces X, Y, and Z, are neglected, the fluid assumed incompressible, and steady state exists, then Equations (1) reduce to

$$\left.\begin{aligned}\frac{\partial p}{\partial x} &= \mu \nabla^2 u \\ \frac{\partial p}{\partial y} &= \mu \nabla^2 v \\ \frac{\partial p}{\partial z} &= \mu \nabla^2 W\end{aligned}\right\} \text{-----------} (2)$$

If the equations of (2) are differentiated with respect to x, y, and z respectively, and added, then the following equation results:

$$\nabla^2 p = 0 \text{-----------} (3)$$

We are now concerned with two dimensional flow only, so Equation (3) may be expressed as

$$\nabla^2 p = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) p = 0 \text{-----------} (4)$$

Equation (3) is now transformed into an equation in bi-polar coordinates. As has been previously mentioned, these coordinates permit boundary conditions which are exceedingly suitable to this problem. No approximations, in regard to film thicknesses, need be made.

The equation which is used to transform from Cartesian coordinates to bi-polar coordinates is

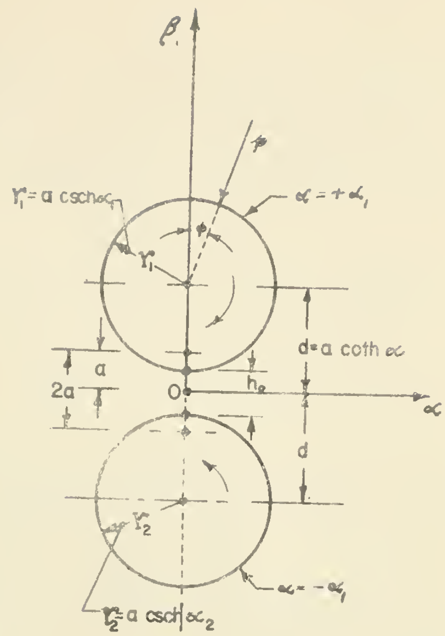


Fig.3 Circular Discs Associated With The Bi-Polar Coordinate System

$$\alpha + i\beta = \log \frac{x + i(y + a)}{x + i(y - a)} \quad \text{.........................(5)}$$

Fig. 3 presents the α, β plane of the bi-polar coordinate system superimposed upon the x, y, plane of the Cartesian system. The origin for both systems is at O. The equation of a circle which represents the circular disc of this problem is as follows:

$$x^2 + (y - a \coth\alpha)^2 = a^2 \mathrm{csch}^2 \alpha \quad \text{........................(6)}$$

The distance (d) from the origin to the center of the circle is given by

$$d = a \coth \alpha \quad \text{..................................(7)}$$

Small a represents the distance from the origin to the focal point of the bi-polar system. The radius (r_1) of the disc is expressed by

$$r_1 = a \operatorname{csch} \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The following relationships exist:

$$x = \frac{a \sin \beta}{\cosh \alpha - \cos \beta} ; y = \frac{a \sinh \alpha}{\cosh \alpha - \cos \beta} \quad \ldots (9)$$

From the conditions of an incompressible fluid we may write

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

We may also write

$$u = -\frac{\partial \psi}{\partial y} ; \quad v = \frac{\partial \psi}{\partial x} ; \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \nabla^2 \psi \text{ ---- } (11)$$

where ψ is the stream function.

In the Cartesian system we may write

$$\zeta = \nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \text{ ---------------- } (12)$$

In the bi-polar system Equation (12) is written as

$$\zeta = h^2 \left(\frac{\partial^2 \psi}{\partial \alpha^2} + \frac{\partial^2 \psi}{\partial \beta^2} \right) \text{ ---------------- } (13)$$

where $(h)^2$ is the reciprocal of the "stretch factor".

It is expressed symbolically by

$$\frac{1}{h^2} = \left(\frac{\partial x}{\partial \alpha}\right)^2 + \left(\frac{\partial y}{\partial \alpha}\right)^2 \text{.........................} (14)$$

or

$$h = \frac{\cosh \alpha - \cos \beta}{a} \text{.................................} (15)$$

The differential equation satisfied by $\nabla^2 \Psi$ is

$$\frac{\partial^2 \zeta}{\partial \alpha^2} + \frac{\partial^2 \zeta}{\partial \beta^2} = 0 \text{ ....} (16)$$

An appropriate solution is (4)

$$\begin{aligned} a\zeta &= 2A_0 + 2B_1 + 2D_0(1 - \cosh\alpha \cos\beta) + 2(B_0 + D_1)(\alpha - \sinh\alpha \cos\beta) \\ &+ 2\sum_{n=1}^{\infty} \Big\{ \Big[(n+1)(A_n + B_{n+1}) - (n-1)(A_{n-1} + B_n)\Big] \cosh n\alpha \cos n\beta \\ &+ \Big[(n+1)(C_n + D_{n+1}) - (n-1)(C_{n-1} + D_n)\Big] \sinh n\alpha \cos n\beta \Big\} \text{ ---} (17) \end{aligned}$$

where the coefficients A_n, B_n, C_n, D_n are arbitrary constants. If we write $h\Psi = \chi$, the corresponding expression for χ may be determined with the aid of the differential equation

(4) See Bibliography

$$a\zeta = (\cosh\alpha - \cos\beta)\left(\frac{\partial^2\chi}{\partial\alpha^2} + \frac{\partial^2\chi}{\partial\beta^2}\right)$$
$$-2\left(\sinh\alpha\,\frac{\partial\chi}{\partial\alpha} + \sin\beta\,\frac{\partial\chi}{\partial\beta}\right)$$
$$+(\cosh\alpha + \cos\beta)\chi \quad \text{----} \quad (18)$$

It is found that

$$\chi = (A_0 + B_0\alpha)\cosh\alpha + (C_0 + D_0\alpha)\sinh\alpha$$
$$+ (A_1\cosh 2\alpha + B_1 + C_1\sinh 2\alpha + D_1\alpha)\cos\beta$$
$$+ \sum_{n=2}^{\infty}\cos n\beta\,\{A_n\cosh(n+1)\alpha + B_n\cosh(n-1)\alpha$$
$$+ C_n\sinh(n+1)\alpha + D_n\sinh(n-1)\alpha\} \quad \text{------} \quad (19)$$

Since the quantities p and $-\mu\zeta$ are conjugate function, equations of the following type are to be satisfied;

$$\frac{\partial p}{\partial\beta} = \mu\frac{\partial\zeta}{\partial\alpha} \quad \text{--------------------} \quad (20)$$

Thus an expression for p in the α, β system is

$$\frac{ap}{\mu} = C - 2D_0\sinh\alpha\sin\beta + 2(B_0 + D_1)(\beta - \cosh\alpha\sin\beta)$$
$$+2\sum_{n=1}^{\infty}\Big\{\big[(n+1)(A_n + B_{n+1}) - (n-1)(A_{n-1} + B_n)\big]\sinh n\alpha\sin n\beta$$
$$+ \big[(n+1)(C_n + D_{n+1}) - (n-1)(C_{n-1} + D_n)\big]\cosh n\alpha\sin n\beta\Big\} \quad \text{--} \quad (21)$$

Equation (21) represents the general expression for the pressure p. If we consider problems in which the pressure is single valued, then all terms possessing α or β alone will not be retained. The coefficient $(B_0 + D_1)$, of β may be set equal to zero.

$$B_o + D_1 = 0 \qquad \text{.......................................(22)}$$

If u_α, u_β are the velocity components in the positive α, and β directions, we have

$$u_\alpha = -h\frac{\partial\Psi}{\partial\beta} = -\frac{\partial\chi}{\partial\beta} + \frac{\Psi}{a}\sin\beta \qquad \text{....................(23)}$$

and

$$u_\beta = -h\frac{\partial\Psi}{\partial\alpha} = \frac{\partial\chi}{\partial\alpha} - \frac{\Psi}{a}\sinh\alpha \qquad \text{.................(24)}$$

We let the angular velocities of the cylinders be represented by ω and write the following equations which represent the boundary conditions:

For $\alpha = +\alpha_1$, $\Psi = \Psi_1$, and $\frac{\partial\chi}{\partial\alpha} = \frac{\Psi_1}{a}\sinh\alpha_1 - a\omega\,\mathrm{cosech}\,\alpha_1$ ....(25)

For $\alpha = -\alpha_1$, $\Psi = -\Psi_1$, and $\frac{\partial\chi}{\partial\alpha} = -\frac{\Psi_1}{a}\sinh(-\alpha_1) - a\omega\,\mathrm{cosech}(-\alpha_1)$ ......(26)

By symmetry for $\alpha = 0$, $\Psi = 0$, and $\frac{\partial\chi^2}{\partial\alpha^2} = 0$ ....(27)

Thus the expression for χ of equation (19) may be reduced, in the case of our problem, to

$$\chi = B_o\alpha(\cosh\alpha - \cos\beta) + C_o\sinh\alpha + C_1\sinh 2\alpha\cos\beta \qquad \text{.......(28)}$$

where

$$\left.\begin{aligned} B_o &= -\frac{a\omega\cosh 2\alpha_1}{2\cosh\alpha_1\sinh^3\alpha_1} \\ C_o &= \frac{a\omega\cosh\alpha_1}{2\sinh^3\alpha_1} \\ C_1 &= -\frac{a\omega}{4\cosh\alpha_1\sinh^3\alpha_1} \end{aligned}\right\} \qquad \text{.........................(29)}$$

If this simplified expression for χ, Equation (28), is substituted into Equation (18), the following expression, appropriate to this problem, may be written:

$$a\gamma = 4C_1\left[\sinh\alpha\cos\beta - 2\sinh\alpha\cosh\alpha\cos^2\beta + \sinh\alpha\cosh\alpha\right] \text{---} (30)$$

The derivative with respect to α yields

$$a\frac{\partial\gamma}{\partial\alpha} = 4C_1\left[\cosh\alpha\cos\beta - 2\cos^2\beta\cosh 2\alpha + \cosh 2\alpha\right] \text{---} (31)$$

Integrating partially with respect to β following the statement of Equation (20) gives

$$ap = C + 4\mu C_1\left[\cosh\alpha\sin\beta - \frac{1}{2}\cosh 2\alpha\sin 2\beta\right] \text{---} (32)$$

Now a combi tion of facts, t hat there is symmetry, that the fluid is assumed incapable of exerting tension stresses, that we are dealing with converging and diverging spaces between the discs. require that the pressure shall pass through zero at the origin where $\beta = \pi$.

This yields $C = 0$. Thus our final expression for the pressure p, in the α, β system, is

$$p = \frac{4\mu C_1}{a}\left(\cosh\alpha\sin\beta - \frac{1}{2}\cosh 2\alpha\sin 2\beta\right) \text{--------} (33)$$

where

$$C_1 = -\frac{a\,\omega}{4\cosh\alpha_1\sinh^3\alpha_1} \text{----------------} (34)$$

An expression for the maximum pressure ($p_{max.}$), along the x-axis or $\alpha = 0$, is

$$p_{Max.} = \frac{3\sqrt{3}\,\mu\, C_1}{a} \qquad \text{.........................(35)}$$

An expression is now obtained for the load-carrying capacity (P) per unit face width of the fluid wedge. We note from Fig. 3. that an expression for this force is

$$P = \int_0^{\pi} r\, p \cos\phi\, d\phi \qquad \text{.........................(36)}$$

It is necessary to express p of Equation 36 in terms of ϕ, remembering that α may be considered constant for any given size disc. Expressions for $\sin\beta$ and $\sin 2\beta$ are the following:

$$\sin\beta = \frac{r \sin\phi \sinh\alpha_1}{d + r\cos\phi} \qquad \text{.........................(37)}$$

$$\sin 2\beta = \frac{r\sin\phi\,(d\sinh 2\alpha_1 + r\sinh 2\alpha_1 \cos\phi - 2a\sinh^2\alpha_1}{(d + r\cos\phi)^2} \text{.(38)}$$

An expression for P is

$$P = \frac{2\mu\, C_1 \sinh\alpha_1}{a}\left(K_1 + K_2\right) \qquad \text{..........(39)}$$

where

$$K_1 = 2\left[2r_1 + d\log\frac{d-r_1}{d+r_1}\right]\left[1 - \cosh 2\alpha_1\right]\cosh\alpha_1 \qquad \text{..........(40)}$$

and $K_2 = 2a \sinh \alpha_1 \cosh 2\alpha_1 \left[\log \frac{d+r_1}{d-r_1} - \frac{2r_1 d}{d^2 - r_1^2} \right]$ .....(41)

We further observe that nearly all the load, carried by the fluid wedge, is distributed over a very small area. Let us arbitrarily define the width of the supporting band of contact x_0 as that distance from the origin, measured along the x-axis, to the point where the pressure drops off to one per cent of its maximum value. An approximate expression for x_0 is

$$x_0 = 6.3\ a \quad \text{.....................................(42)}$$

From previous work by this author (5) it was found that an approximate solution to this problem gave the following set of equations:

<u>For the pressure p</u>

$$p = G\left(\frac{1}{24} \sin 4S + \frac{1}{12} \sin 2S \right) \quad \text{..............(43)}$$

where $$G = \frac{12 \mu_0 U \sqrt{2 r h_0}}{h_0^2} \quad \text{..............(44)}$$

U is the peripheral velocity of the disc.

$$r = \frac{r_1 r_2}{r_1 + r_2} \quad \text{For equal size discs } r = \frac{r_1}{2} \quad \text{.....(45)}$$

$$x = \sqrt{2 r h_0}\ \tan S \quad \text{.....(46)}$$

(5) See Bibliography

<u>For the maximum pressure $p_{max.}$</u>

$$p_{max.} = \frac{\sqrt{3}}{16} G \text{ ------------------------------ } (47)$$

For the load-carrying capacity P

$$P = \frac{G}{6}\sqrt{2rh_o} \text{ ------------------------- } (48)$$

<u>For the width of the band of contact</u>

$$x_o = \sqrt{2rh_o}\,\tan(80^\circ 58') \text{ ----------------- } (49)$$

We shall now apply these equations in analyzing a given problem. Since the exact solution is obtained for the case of symmetrical flow only, we shall take discs of equal size revolving at the same speed ω. The discs are 3.0000 inches in diameter. Their angular velocity is (-183) rad/sec. The minimum thickness of the wedge is h_o = 0.00002[illegible] inches. The viscosity of the oil is $\mu = 5.5 \times 10^{-6}$ lb sec/sq.in. We shall compute the following by the exact and approximate methods and compare the results:

1. The maximum pressure within the wedge
2. The load-carrying capacity P of the wedge per inch of face width
3. The width x_c of the supporting band of contact

<u>SOLUTION OF PROBLEM: - EXACT METHOD</u>

Equation (35) is the expression for the maximum pressure

$$p_{max.} = \frac{3\sqrt{3}\,\mu\, C_1}{a} \text{ -------------------- } (50)$$

$$\frac{C_1}{a} = -\frac{\omega}{4\cosh\alpha_1 \sinh^3\alpha_1} \text{ -------------- } (51)$$

To find α_1 we need to use the following:

$$d_1 = a \coth \alpha_1 \; ; \quad \rho = a \operatorname{csch} \alpha_1$$

$$\text{or} \quad \frac{d}{\rho} = \cosh \alpha_1$$

$d = 1.5000 + 0.00001$[redacted] $= 1.50001$[redacted] inches

$$\text{so} \cosh \alpha_1 = 1.000006667$$

$$\alpha_1 = 0.00365 \text{ rad.}$$

and $a = \rho \sinh \alpha_1 = 1.5000 \times 0.00365$

$$a = 0.00547$$

Thus

$$p_{MAX.} = 3\sqrt{3} \times 5.5 \times 10^{-6} \left[\frac{183}{4 \times 1.000006667 \times 0.00365^3} \right] \text{----} (52)$$

$$p_{MAX.} = +27{,}600 \text{ psi} \text{ ------------------ } (53)$$

Equation (39) is the expression for the load-carrying capacity P

$$P = \frac{2\mu\, C_1 \sinh \alpha_1}{a} (K_1 + K_2) \text{ --------- } (54)$$

$$K_1 = +0.00084$$
$$K_2 = -5.97$$

$P = -226$ lb in. of face width .... (55)

Equation (42) is an expression for the width of the supporting band of contact

$x_0 = 6.3a = 6.3 \times 0.00547 = 0.0344$ inches ....(56)

SOLUTION OF PROBLEM: - APPROXIMATE METHOD

The maximum pressure $p_{max.}$ is given by

$$p_{max.} = \frac{\sqrt{3}}{16} G = \frac{\sqrt{3}}{16} \times 247{,}000 = 26{,}700 \text{ psi}$$

The load-carrying capacity P is indicated by

$$P = \frac{G}{6}\sqrt{2rh_0}$$

P = 225 lbs. per inch of face width.

The width of the band of contact is

$$x_0 = \sqrt{2rh_0} \times \tan(80°58') = 0.034''$$

CONCLUSION CONCERNING STEADY STATE SOLUTION

We find that the approximate solution derived earlier, gives results which are close to the exact answers. However, at this point, our results are of little account in the solution of the usual non-steady-state problem except for a guide in the solution of steady state problems. If we wish to get practical results for these problems in hydrodynamics we are forced to get the solution of these non-steady-state problems through experimental analyses. We now proceed to Part two of this report.

PART II

INTRODUCTION:

The thickness and the load supporting capacity of oil films entrapped between circular rotating discs on parallel shafts were investigated here by experimental methods. (see Fig. 1E). The fluid wedge was formed by dragging the oil between the discs which were rotated in opposite directions. The wedge was made alternately thick and thin by vibrating the beam which supported the lower shaft. The D.C. excitation coil set up lines of flux which had to pass through the neutral gap which separated the surfaces of the two sets of double discs. The pick-up coil delivered a signal to the galvanometer oscillograph as the lower shaft was vibrated. An automatic camera was used to photograph the oscillograms. Oscillograms were taken with and without fluid wedges between the discs. The presence of oil caused the beam supporting the lower shaft to vibrate through relatively small amplitudes. Small lead wires of the order of 0.020 inches were rolled between the two sets of discs with the lower beam supported by jacks to prevent bending. The thickness of the neutral gap h_o was measured by obtaining the thickness of the flattened lead wire. The neutral gap thickness h_o was adjusted until the amplitude of the vibrating beam, for a power setting of the amplifier of 60, equaled the value of the gap h_o.

The constant D.C. excitation current for the pick-up system came from two storage batteries. The D C. and A.C.

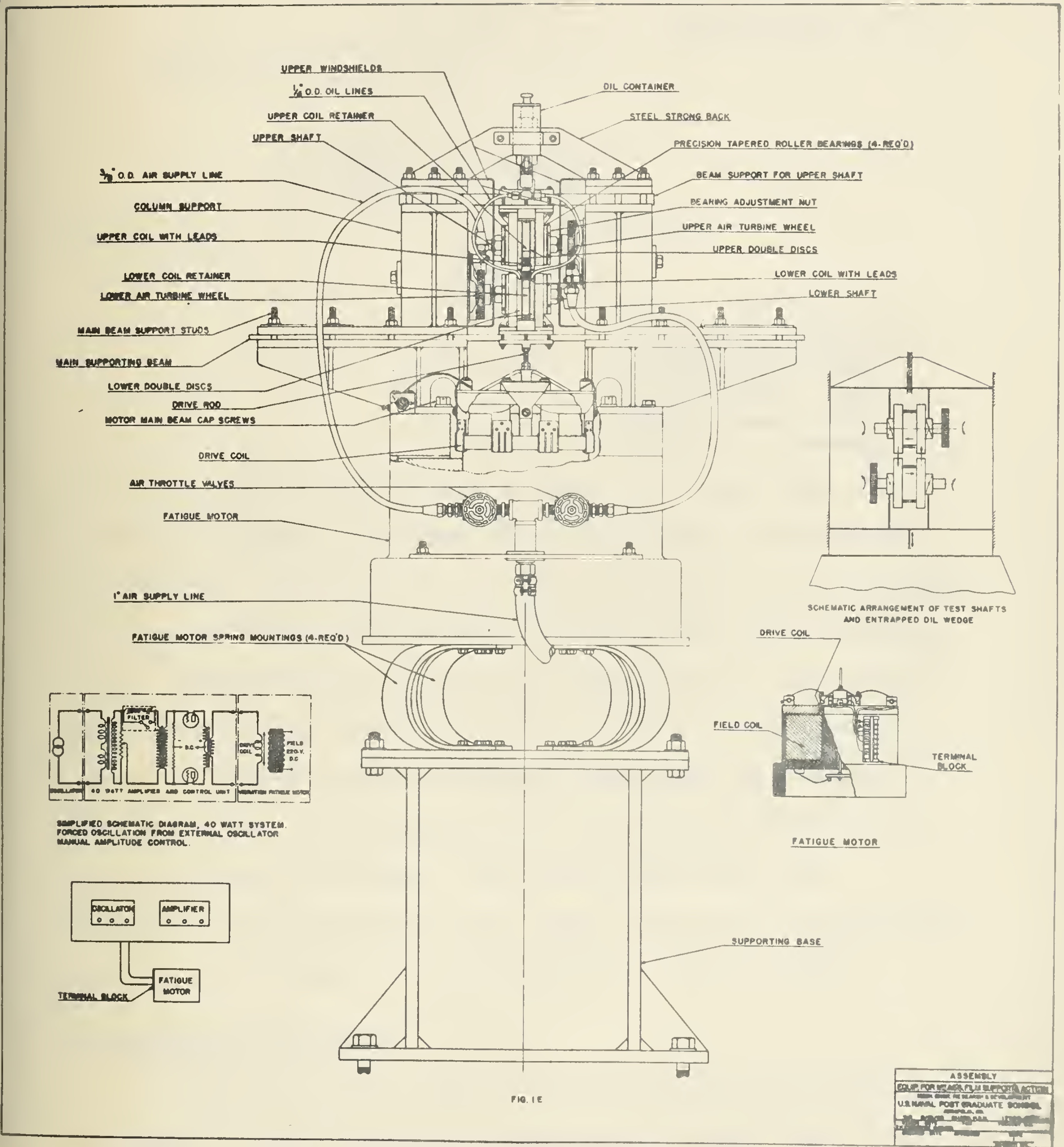
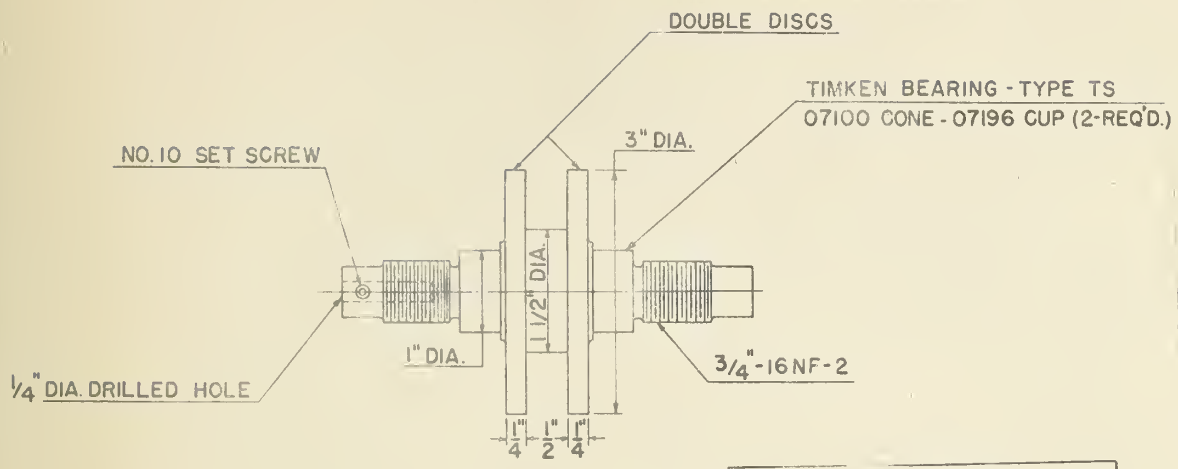

UPPER WINDSHIELDS
1/4" O.D. OIL LINES
UPPER COIL RETAINER
UPPER SHAFT
3/16" O.D. AIR SUPPLY LINE
COLUMN SUPPORT
UPPER COIL WITH LEADS
LOWER COIL RETAINER
LOWER AIR TURBINE WHEEL
MAIN BEAM SUPPORT STUDS
MAIN SUPPORTING BEAM
LOWER DOUBLE DISCS
DRIVE ROD
MOTOR MAIN BEAM CAP SCREWS
DRIVE COIL
AIR THROTTLE VALVES
FATIGUE MOTOR
1" AIR SUPPLY LINE
FATIGUE MOTOR SPRING MOUNTINGS (4-REQ'D)
OIL CONTAINER
STEEL STRONG BACK
PRECISION TAPERED ROLLER BEARINGS (4-REQ'D)
BEAM SUPPORT FOR UPPER SHAFT
BEARING ADJUSTMENT NUT
UPPER AIR TURBINE WHEEL
UPPER DOUBLE DISCS
LOWER COIL WITH LEADS
LOWER SHAFT
SCHEMATIC ARRANGEMENT OF TEST SHAFTS AND ENTRAPPED OIL WEDGE
DRIVE COIL
FIELD COIL
TERMINAL BLOCK
FATIGUE MOTOR
FILTER
D.C.
FIELD 220-V. D.C.
40 WATT AMPLIFIER AND CONTROL UNIT
SIMPLIFIED SCHEMATIC DIAGRAM, 40 WATT SYSTEM.
FORCED OSCILLATION FROM EXTERNAL OSCILLATOR
MANUAL AMPLITUDE CONTROL.
OSCILLATOR
AMPLIFIER
FATIGUE MOTOR
TERMINAL BLOCK
SUPPORTING BASE
ASSEMBLY
U.S. NAVAL POST GRADUATE SCHOOL

FIG. 1E



SHAFT (2-REQ'D.)

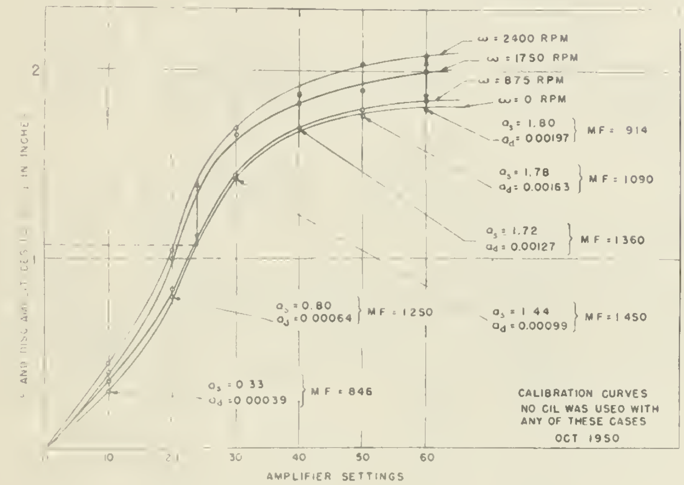
FIG. 2E

currents and voltages to the fatigue motor were fixed for various amplifier settings. Then the values of the height of the wave form on the scope were found for various settings of the amplifier with the angular velocities of the discs at zero. These values are plotted in Graph No. 1. Lead wires were then rolled between the discs, left in position with $\omega = 0$, and the vibrator pounded them until the height of the scope wave form equaled that which existed for the case of air between the discs. Leads were taken and their thickness measured for all six power settings of the amplifier, except no. 50 where the leads became too thin to handle.

EXPERIMENTAL RESUDTS:

It is clear now that at this point we have information at hand for determining the magnification factor (M.F.) of

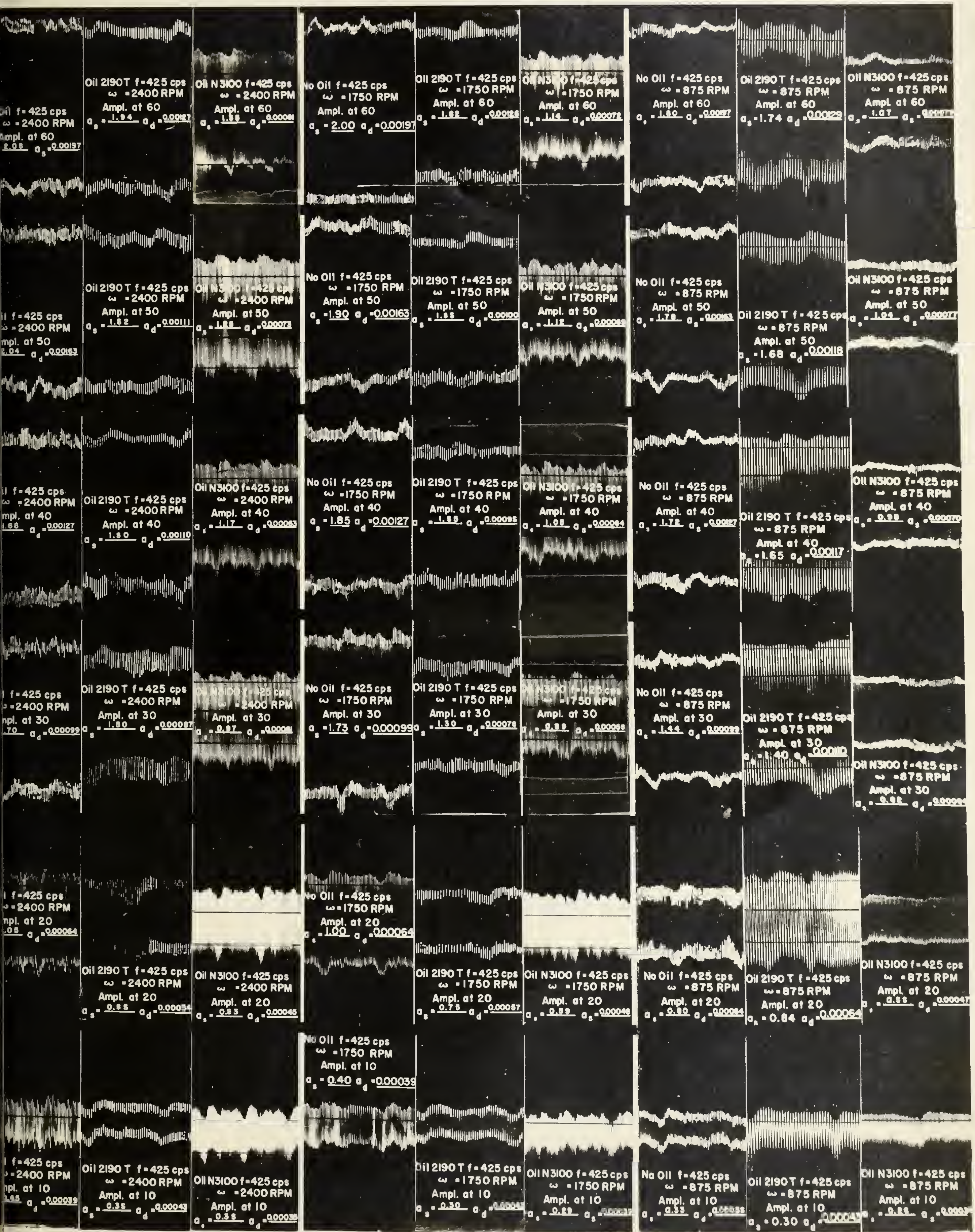
our set-up. For example, we find from Graph No. 1, that for an amplifier setting of 60 (without oil and $\omega = 0$) the lower disc vibrated through an amplitude of $h_o = 0.00197$ inches, while the height of the scope wave form was 1.80 inches. This gives a M.F. = 914. Likewise, when the amplifier



GRAPH NO 1

was set at 30 the leads showed that the lower shaft vibrated through $a_d = 0.00099$ inches, while the scope gave a_s = 1.44 inches. The M.F. for this setting was 1450.

It was noticed that the a_s depended not only upon the amplitude of vibration of the disc a_d, but also upon the speed with which the discs rotated. It thus became necessary to plot the scope amplitude against the amplifier settings for various values of the angular velocity ω. These


il f=425 cps
ω =2400 RPM
Ampl. at 60
Oil 2190 T f=425 cps
ω =2400 RPM
Ampl. at 60
a_s = 1.94 a_d =0.00127
Oil N3100 f=425 cps
ω =2400 RPM
Ampl. at 60
No Oil f=425 cps
ω =1750 RPM
Ampl. at 60
a_s = 2.00 a_d =0.00197
Oil 2190 T f=425 cps
ω =1750 RPM
Ampl. at 60
Oil N3100 f=425 cps
ω =1750 RPM
Ampl. at 60
a_s = 1.14 a_d =0.00072
No Oil f=425 cps
ω =875 RPM
Ampl. at 60
Oil 2190 T f=425 cps
ω =875 RPM
Ampl. at 60
a_s=1.74 a_d=0.00129
Oil N3100 f=425 cps
ω =875 RPM
Ampl. at 60
il f=425 cps
ω =2400 RPM
mpl. at 50
Oil 2190 T f=425 cps
ω =2400 RPM
Ampl. at 50
a_s = 1.52 a_d =0.00111
Oil N3100 f=425 cps
ω =2400 RPM
Ampl. at 50
No Oil f=425 cps
ω =1750 RPM
Ampl. at 50
a_s =1.90 a_d =0.00163
Oil 2190 T f=425 cps
ω =1750 RPM
Ampl. at 50
Oil N3100 f=425 cps
ω =1750 RPM
Ampl. at 50
No Oil f=425 cps
ω =875 RPM
Ampl. at 50
Oil 2190 T f=425 cps
ω =875 RPM
Ampl. at 50
a_s =1.68 a_d =0.00118
Oil N3100 f=425 cps
ω =875 RPM
Ampl. at 50
il f=425 cps
ω =2400 RPM
mpl. at 40
Oil 2190 T f=425 cps
ω =2400 RPM
Ampl. at 40
Oil N3100 f=425 cps
ω =2400 RPM
Ampl. at 40
No Oil f=425 cps
ω =1750 RPM
Ampl. at 40
a_s =1.85 a_d =0.00127
Oil 2190 T f=425 cps
ω =1750 RPM
Ampl. at 40
Oil N3100 f=425 cps
ω =1750 RPM
Ampl. at 40
No Oil f=425 cps
ω =875 RPM
Ampl. at 40
Oil 2190 T f=425 cps
ω =875 RPM
Ampl. at 40
a_s =1.65 a_d =0.00117
Oil N3100 f=425 cps
ω =875 RPM
Ampl. at 40
l f=425 cps
=2400 RPM
npl. at 30
Oil 2190 T f=425 cps
ω =2400 RPM
Ampl. at 30
Oil N3100 f=425 cps
=2400 RPM
Ampl. at 30
No Oil f=425 cps
ω =1750 RPM
Ampl. at 30
a_s =1.73 a_d =0.00099
Oil 2190 T f=425 cps
ω =1750 RPM
Ampl. at 30
Oil N3100 f=425 cps
ω =1750 RPM
Ampl. at 30
No Oil f=425 cps
ω =875 RPM
Ampl. at 30
Oil 2190 T f=425 cps
ω =875 RPM
Ampl. at 30
a_s =1.40 a_s =0.00110
Oil N3100 f=425 cps
ω =875 RPM
Ampl. at 30
l f=425 cps
=2400 RPM
npl. at 20
No Oil f=425 cps
ω=1750 RPM
Ampl. at 20
a_s =1.00 a_d =0.00064
Oil 2190 T f=425 cps
ω =2400 RPM
Ampl. at 20
Oil N3100 f=425 cps
ω =2400 RPM
Ampl. at 20
Oil 2190 T f=425 cps
ω =1750 RPM
Ampl. at 20
Oil N3100 f=425 cps
ω =1750 RPM
Ampl. at 20
No Oil f=425 cps
ω =875 RPM
Ampl. at 20
Oil 2190 T f=425 cps
ω=875 RPM
Ampl. at 20
a_s = 0.84 a_d =0.00064
Oil N3100 f=425 cps
ω =875 RPM
Ampl. at 20
No Oil f=425 cps
ω =1750 RPM
Ampl. at 10
a_s = 0.40 a_d =0.00039
l f=425 cps
=2400 RPM
npl. at 10
Oil 2190 T f=425 cps
ω =2400 RPM
Ampl. at 10
Oil N3100 f=425 cps
ω =2400 RPM
Ampl. at 10
Oil 2190 T f=425 cps
ω =1750 RPM
Ampl. at 10
Oil N3100 f=425 cps
ω =1750 RPM
Ampl. at 10
No Oil f=425 cps
ω =875 RPM
Ampl. at 10
Oil 2190 T f=425 cps
ω =875 RPM
Ampl. at 10
a_s = 0.30 a_d =0.00043
Oil N3100 f=425 cps
ω =875 RPM
Ampl. at 10

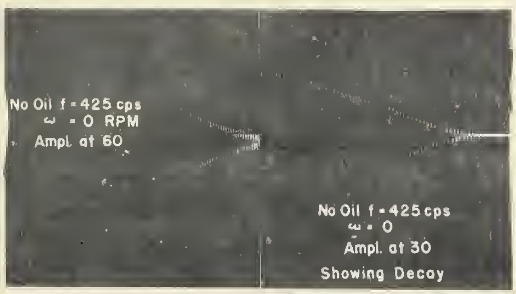
values are shown in Graph No. 1. The magnification factor M.F. will always be determined with reference to the graph of ω = 0. The increase in the amplitude of the scope wave due to the rotation of the discs is nearly nullified for the cases with and without oil, since the angular velocity was fixed.

We indicate now how one may use Graph No. 1 to determine the load carried by a certain thickness oil film wedge. From Table No. 1 we find the following: that the height of the scope wave form, corresponding to ω = 2400 RPM without oil with the amplifier at 60, is 2.08 inches, that the amplitude of the scope wave form, corresponding to ω = 2400 RPM with N3100 oil with the amplifier at 60, is 1.35 inches. We note that 0.28 inches of the scope amplitude are due to rotational effects. ow the amplitude of vibration of the lower disc, without oil, with the amplifier at 60 and f = 425 cps is just sufficient to close the neutral gap setting of h_o = 0.00197 inches. Thus, for the case of no oil, with ω = 2400 RPM, with f = 425 cps, the M.F. is found to be 1.80/0.00197 or 914. The magnification factor is defined as the ratio of a_s/a_d. This value is calculated with reference to the curve ω = 0. For the case of the use of N3100 oil with ω = 2400 RPM, with f = 425 cps we find from Table one that the a_s = 1.35 inches. We recall that 0.28 inches of this amplitude is due to rotational effects, thus the value corrected to the ω = 0

TABLE No. I — Nov. 1950

	ω = 2400 RPM Temp. 70° F.													
	No Oil	2190 T						N 3100						No Oil
AMPLIFIER SETTINGS	a_s INS.	a_s INS.	a_d INS.	M.F.	t INS.	F_o lb.	$F_{s.s.}$ lb.	a_s INS.	a_d INS.	M.F.	t INS.	F_o lb.	$F_{s.s.}$ lb.	P_o lb.
60	2.08	1.94	0.00127	1360	0.00070	140	20	1.35	0.00081	914	0.00116	162	73.3	102.6
50	2.04	1.82	0.00111	1420	0.00086	140		1.25	0.00073	1280	0.00124	158		98.4
40	1.88	1.80	0.00110	1420	0.00087	140		1.17	0.00063	1268	0.00134	156		93.6
30	1.70	1.50	0.00087	1350	0.00110	117		0.97	0.00061	1156	0.00136	134		81.9
20	1.05	0.88	0.00054	1146	0.00143	69		0.63	0.00045	976	0.00152	74		48.9
10	0.45	0.39	0.00043	635	0.00154	21		0.36	0.00035	622	0.00162	22		20.4
	ω = 1750 RPM													
	No Oil	2190 T						N 3100						No Oil
AMPLIFIER SETTINGS	a_s INS.	a_s INS.	a_d INS.	M.F.	t INS.	F_o lb.	$F_{s.s.}$ lb.	a_s INS.	a_d INS.	M.F.	t INS.	F_o lb.	$F_{s.s.}$ lb.	P_o lb.
60	2.00	1.82	0.00126	1341	0.00071	136	15	1.14	0.00072	1255	0.00125	164	53.3	102.6
50	1.90	1.69	0.00100	1445	0.00097	142		1.12	0.00069	1252	0.00128	164		98.4
40	1.85	1.65	0.00099	1450	0.00098	132		1.05	0.00064	1250	0.00133	154		93.6
30	1.73	1.30	0.00076	1309	0.00120	123		0.89	0.00058	1214	0.00139	133		81.9
20	1.00	0.75	0.00057	1060	0.00140	68		0.59	0.00046	976	0.00151	73		48.9
10	0.40	0.30	0.00043	620	0.00154	20		0.29	0.00039	564	0.00158	20		20.4
	ω = 875 RPM													
	No Oil	2190 T						N 3100						No Oil
AMPLIFIER SETTINGS	a_s INS.	a_s INS.	a_d INS.	M.F.	t INS.	F_o lb.	$F_{s.s.}$ lb.	a_s INS.	a_d INS.	M.F.	t INS.	F_o lb.	$F_{s.s.}$ lb.	P_o lb.
60	1.80	1.74	0.00129	1336	0.00068	136	7.3	1.07	0.00079	1250	0.00118	164	26.7	102.6
50	1.78	1.68	0.00118	1405	0.00079	134		1.04	0.00077	1320	0.00120	154		98.4
40	1.72	1.65	0.00117	1404	0.00080	133		0.95	0.00070	1262	0.00127	150		93.6
30	1.44	1.40	0.00110	1320	0.00087	105		0.82	0.00064	1250	0.00133	154		81.9
20	0.80	0.84	0.00064	1250	0.00133	65		0.55	0.00047	1046	0.00150	73		48.9
10	0.33	0.30	0.00043	620	0.00154	20		0.26	0.00037	560	0.00160	20		20.4

DYNAMIC SP CONSTANT OF BEAM = 250,000 lbs / IN.
VOLTAGE TO VIBRATION MOTOR = 8 V (ACCORDING TO LOAD).
CURRENT TO VIBRATION MOTOR = 200 M.A. (ACCORDING TO LOAD).
VOLTAGE TO VIBRATION MOTOR = 118 V D.C.
CURRENT TO VIBRATION MOTOR = 0 9 A. D.C.
VOLTAGE TO PICK-UP SYSTEM = 12 V. D.C.
CURRENT TO PICK-UP SYSTEM = 0 25 A D.C.
$t = h_o - a_d$ MINIMUM OIL WEDGE THICKNESS.
TOTAL WT. ON BEAM = 12.11 lbs
NATURAL FREQ OF BEAM WITH WT = 12.11 lbs = 450 cps
F_o = LOAD CARRYING CAPACITY OF FILM IN lbs/IN. OF FACE WIDTH.
$F_{s.s.}$ = LOAD CARRYING CAPACITY OF FILM (FROM STEADY STATE SOLUTION)
P_o = AMPLITUDE OF MOTOR OUT PUT FORCE.



curve, is 1.07 inches. This gives a M.F. = 1326. This scope amplitude of 1.07 inches on the $\omega = 0$ curve corresponds to an amplifier setting of 24, and thus a disc amplitude of 0.00081 inches. The amplitude of vibration of the beam was thus reduced from h_o = 0.00197 inches without oil, to 0.00081 inches with oil. We know now that the minimum oil film wedge thickness was 0.00116 inches. The load on the film will be computed now.

From the mechanics of the problem we observe that the beam may be regarded as massless with an equivalent weight placed at the center of the span. There is also the weight of the lower shaft, bearings, hangars, motor drive coil, etc. which may be considered concentrated at the midsection of the beam. This total weight = 12.11 lbs. We see that four forces act on the beam in the displaced position, namely; the beam restoring force F_b, the force of internal damping F_d, the force of the oil film wedge F_o, and the force of the vibration motor F_m. The amplitude and the phase relationship of these forces are shown in Fig. 3E. The sum of the vertical components may be written as:

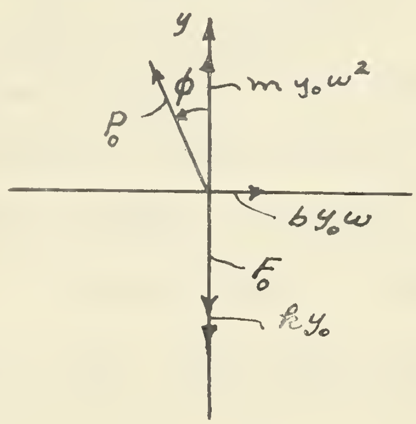


FIG. 3E

$$m\omega^2 y_o + P_o \cos\Phi - F_o - k y_o = 0 \qquad \text{............(57)}$$

The value of P_o, the amplitude of the motor output force, was found for different amplifier settings, by placing SR4 strain gages on a specially prepared drive rod. These values are given in Table No. 2. It was found that the amplitude of the motor output force P_o depended upon the amplifier setting for a given frequency. Thus the motor output force was evaluated without the use of oil between the discs.

The phase angle Φ is shown to be

$$\tan \Phi = \frac{2 \frac{b}{b_{cr}} \cdot \frac{\omega}{\omega_n}}{1 - (\omega^2/\omega_n^2)} \qquad \text{.........(58)}$$

where b is the damping constant of the system and is evaluated from the die-away wave form shown in the preceding photograph. It was found that b = 0.95 lb. sec./ in. The value of the critical damping constant $b_{cr} = \sqrt{4mk}$ was found to be b_{cr} = 177 lb. sec./ in. The weight on the beam was 12.11 lbs. and the dynamic spring constant was 250,000 lbs./in. The undamped natural frequency of the system was ω_n = 2830 rad./ sec. It was found that $\Phi = 5^\circ - 10^1$.

The amplitude of load F_o carried by the oil film wedge may now be found from Eq. 57. The disc or beam amplitude of vibrations a_d or y_o was found to be 0.00081". The mass m on the beam was 0.0314 $\frac{\text{lbs. sec.}^2}{\text{in.}}$. The frequency of vibration of the beam was 425 cps or 2670 rad./ sec. The value of P_o

was 102.61 lbs. Thus the oil wedge, the minimum thickness of which was 0.00116 ", carried a load of

$$F_o = -250{,}000 \times 0.00081 + 102.6 \times 0.996 + 0.0314 \times \overline{2670}^{2} \times 0.00081 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(59)$$

or

$$F_o = 81.00 \text{ lbs.} / \text{half inch face width} \quad \ldots\ldots\ldots\ldots\ldots(60)$$

$$F_o = 162.0 \text{ lbs.} / \text{inch of face width} \quad \ldots\ldots\ldots\ldots(61)$$

If one used the results of Part I (Eq. 56) to predict the width of the supporting band of contact x_o , then the average pressure within the film will be

$$P_{av.} = \frac{F_o}{A} = \frac{81.0}{0.5 \times 0.0344} = 4710 \text{ psi.} \quad (62)$$

One may count on the maximum pressure being approximately seven times the average. This would give a maximum pressure of

$$p_{max} = 33{,}000 \text{ psi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(63)$$

It will now be of considerable interest to compute the load which this same minimum thickness oil film wedge will support on the basis of steady state conditions. Eq. 39 or Eq. 48 may be used to obtain this result. In Eq. 48

$$G = \frac{12 \mu_a \; U \sqrt{2rh_o}}{h_o^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(64)$$

$\mu_a = 1.29 \times 10^{-4}$ reyns (N3100 - 70°F).

U = 378" / sec. (2400 RPM - 3" diam.disc)

$h_o = 0.00197$

G = 8130 psi

Thus

$$P = \frac{G}{6} \sqrt{2rh_o}$$

$$= \frac{8130}{6} \times 0.054 = 73.3 \text{ lbs.}/ \text{ in. of face width or}$$

$$P = 36.6 \text{ lbs.}/ \text{ half inch of face width.}$$

This is the load which the oil wedge could be expected to carry on the basis of steady state conditions. It is observed that the non-steady state solution indicates that this same wedge can carry 81 lbs. The non-steady state solution then shows that the wedge can carry 2.22 times as much load as the steady state solution indicates. From an inspection of Fig. 4E it will be observed further that the ratio of the largest value of F_o to the smallest value of F_{ss} is $\frac{162}{27} = 6$. This means that the load supporting action of the film, resulting from the approach motion of the metal bounding surfaces, can be six times as large as that resulting from the rotational effects of the discs. In other words the non-steady state solution gives values which are six times as large as those of the steady state. This is a factor which we may no longer omit in our predictions of the load carrying capacity of films. It is no wonder that we have not been able to account for the large loads which bearings and gears actually carry. Our theory is based on steady state conditions.

The preceding point is the prominent part of this whole

research project. The oil film wedge in bearings and gears supports loads because they operate under non-steady state conditions.

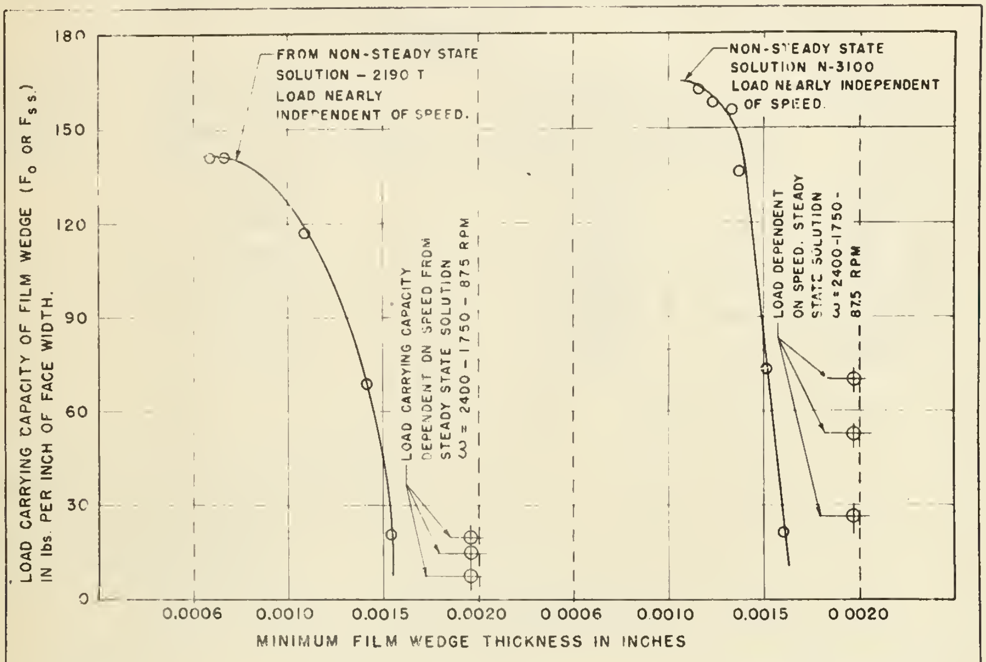


FIG. 4 E

Information on use of strain gages:

The amplitude of the motor output force was determined through the use of strain gages. Two A-7 strain gages were used on the flat section of the drive rod. A third temperature compensating gage was employed. The strain indication was received on a cathode ray oscillograph, through the employment of a Baldwin Southwark strain calibration and amplifier instrument. Two gages were used so that bending

of the drive rod could be detected. It was determined that a strain of 120 micro inches per inch corresponded to an amplitude ordinate height of 0.7 inches on the cathode ray oscillograph with the gain as used. The cross-sectional area of the steel drive rod was A = 0.0100 sq. ins.

TABLE II

Amplifier settings	Amplitude Gage A in inches	Amplitude Gage B in inches	P_o lbs.
60	1.99	2.00	102.6
50	1.90	1.91	98.4
40	1.80	1.81	93.6
30	1.58	1.59	81.9
20	0.95	0.95	48.9
10	0.35	0.40	20.4

Information on oils used:

N3100 - navy symbol:

specific gravity at 60 / 60°F = 0.903

A.P.I. degrees = 25.2

Viscosity seconds, S U V, at 100°F = 1318

" " " " 130°F = 492

" " " " 210°F = 90.9

Viscosity index (Dean and Davis) = 72.9

2190T- turbine oil

specific gravity at 60/ 60°F = 0.8844

A.P.I. degrees = 28.5

Viscosity seconds, S U V, at 100°F = 420

" " " " 130°F = 193

" " " " 210°F = 58

(Engineering Experiment Station Test 1 BDT031)

Additional Remarks

During the operation of the test equipment, it was found that an optional method of measuring the value of h_o gave a good check on those values already found through the use of leads. All these test results are reproducible.

In the test progra m to date the value selected for the netitral gap setting (h_o = 0.00197") was relatively large so that the beam would, in effect, vibrate about its static neutral equilibrium position, rather than about some displaced position. In future work, if small values of h_o are used, it will be necessary to correct the film thickness to account for this fact. The material in the steel discs was SAE 1045 steel hardened to 45 RC. The surface roughness was 20 millionths of an inch.

The irregular frequency components of vibration present in the various wave forms shown in the photograph result from various causes such as non-concentricity of the discs, non-uniformity of the rollers in the bearings, and non-uniform speed of the discs. These components do not alter appreciably the results of the test.

Conclusions:

A prominent characteristic of the load supporting capacity of fluid films has been uncovered in this research work. It is definitely shown that thin fluid films strongly resist being squeezed out from between metal surfaces,

especially if the bounding surfaces are approaching each other at high velocities. In other words, the non-steady-state character of the motion of the bounding surfaces of fluid films is responsible for their ability to support relatively great loads.

Our steady state theory never has been able to account satisfactorily for the relatively large loads which our bearings and gears carry every day. This theory indicates unreasonably thin film thicknesses for these members when used with heavy loads at low speeds; many times these thicknesses are much smaller than the surface roughness of the metal bounding surfaces. Neither can the theory account for the loads which are carried successfully in very high speed gearing such as in certain turbine gear units. In such cases the period of mating is very short, and the whole lubrication phenomenon is non-steady-state, without question. It is highly probable that the prominent point uncovered in this work partly explains why such gears, actually work.

Two external type cylindrical discs, which formed the metal bounding surfaces of the fluid film under investigation, were chosen in this work in preference to actual bearings, because of their basic character in relation to gears as well as to bearings. The results of this work may be used in studies of gears as well as in internal

and external bearings. For one of the cases examined in this work, the non-steady-state figures collected from actual experimental tests indicated that the film under investigation carried a load six times as large as the steady state solution would have indicated.

No attempt will be made at present to formulate a non-steady-state theory to explain this non-steady-state phenomenon. But it will not be difficult to do this once we have more test data to draw from in forming semi-empirical theories. We should consider this work a simple beginning in the investigation of a most complex problem.

It has been the opinion of this author for some time that theory alone is an insufficient tool for completely explaining some of these complex lubrication phenomena. Thus we at the Naval Postgraduate School have spent the last three years in acquiring and testing accurate instruments for carrying out this mission. It cannot be overemphasized that we have simulated actual operating conditions. We have worked with actual steel discs with real lubricants. These discs are deformable; they have surface roughness; they have certain adhesive and cohesive properties. The lubricants act in their natural fashion. Above all we have worked with non-steady-state conditions.

This problem should be pursued vigorously along these combined experimental and theoretical lines until

enough date is gathered so that we may formulate the proper semi-empirical theories to give important guidance to designers in the fields of bearing and gears as well as in other related fields.

Indebtedness:

The author is indebted to Lt. Commander A. Ahrens and several colleagues in the mathematics, physics and mechanical engineering departments for their assistance. The machine shops of the U. S. Naval Postgraduate School, the Engineering Experiment and the Naval Gun Factory cooperated fully.

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